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CALCULATION OF HEAT TRANSFER DURING WATER FLOW IN PROFILED TWISTED PIPES

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UDC 536.242

A Semiempirical model of heat exchange during water flow in profiled twisted pipes is formulated on the basis of the modified Prandtl–Taylor analogy.

It is well known [1-7] that a rigorous analytic investigation of hydrodynamics and heat exchange in pipes and channels with artificial roughness is practically impossible, which is determined by the extremely complicated flow structure. This situation also pertains fully to profiled twisted pipes (PTP), where the interaction of axial, swirled, and separation flows is observed. The relations between the intensities of each of these flows, the boundaries of which are practically impossible to predict because of their mutual overlapping, are evidently determined by the geometry of the PTP and by the flow regime of the axial stream. It must be emphasized that near the pipe wall the flow is three-dimensional: The stream has velocity components along the knurling (resulting in friction of the stream against the pipe wall) and perpendicular to the projections (resulting in the loss of mechanical energy in the developing vortices), as well as a radial velocity component. The relations between the velocity components of these flows, and hence the share of friction and local resistances in the overall energy dissipation, can be estimated only by modeling. From the data of a number of papers [3, 4, 6] it has been established that right at the projections (in a zone of artificial roughness) the flow has a cellular character – horseshoe-shaped vortices are formed, the dynamics of which depend essentially on the shape and size of the roughness. Moreover, as noted in [8], the flow region in the zone above the projections is filled with vortices of different scales. Under these conditions, the construction of a calculating model of the flow requires a certain schematization.

On the basis of the concepts presented above, with allowance for the results of investigations of the hydrodynamics of water flow in glass PTP, a semiempirical model of the process of heat exchange during the flow of a one-phase heat-transfer agent in a PTP is based on the modified Prandtl–Taylor analogy. The main idea of this analogy consists in the summing of the thermal resistances of the different regions through which the heat flow occurs [8]. For a two-layer Prandtl–Taylor model the total thermal resistance consists of the sum of the thermal resistances of the regions of turbulent (R_t) and molecular (R_m) transfer:

$$R = R_t + R_m. \tag{1}$$

The following equation was obtained in [8] for moderate Prandtl numbers, using the generalized Reynolds analogy for the thermal resistance in a fully turbulent region:

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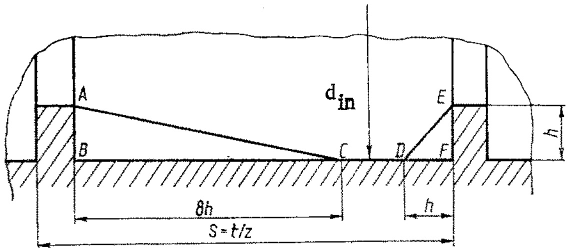

Fig. 1. Diagram of wall flow of a heat-transfer agent lying at the foundation of the model: ABC, DEF) vortex zone I; CD) zone of relatively smooth twisted flow II.

$$\frac{\lambda_{\text{pr}}}{8\,\text{St}} = \text{Pr}_t\left(1 - \frac{V_s}{V_f}\right) + \frac{\tau_w C_p}{V_f} R_m. \tag{2}$$

We assume that the thickness of the viscous sublayer on the "smooth" section between projections of the knurling is considerably less than the characteristic size h of a projection. Then the thermal resistance R_m of the wall zone is determined entirely by the hydrodynamics of flow in the space between projections, i.e., the regime of "developed roughness" occurs. In this case, the transitional zone between the vortical wall region and the external, fully developed turbulent flow is absent, i.e., $(V_s/V_f) \ll 1$ [8]. Then Eq. (2) takes the form

$$\frac{\lambda_{\text{pr}}}{8\text{St}} = \text{Pr}_t + \frac{\tau_w C_p}{V_f} R_m. \tag{3}$$

The problem consists in giving R_m the value characterizing the intensity of heat transfer in the wall zone of a PTP.

The scheme of wall flow of a heat-transfer agent in a PTP is analyzed, by analogy with [5], in the following form (Fig. 1). The space between two adjacent projections (turbulizers), spaced over a distance S, is occupied by a vortex zone I (ABC and DEF) with a total length of 9h along the wall and by a zone of relatively smooth twisted flow II (CD). As was shown in [5], the quasisteady vortex zone is practically insensitive to the orientation of the projection relative to the stream of heat-transfer agent in a wide range of variation of the onflow angles. It is assumed that S > 9h, while the thickness (width) of the projection (turbulizer) is small and it can be neglected. The heat flux from the wall to the stream of heat-transfer agent in the PTP is determined by the sum of the heat fluxes in zones I and II,

$$q = q_{\text{I}} + q_{\text{II}}, \tag{4}$$

where q_{I} is the heat flux through the vortex region; q_{II} is the heat flux through the zone of "smooth" flow.

The area of contact of the vortex region with the pipe surface, within the limits of the knurling pitch along a filament, is

$$F_{\text{vort}} \simeq 9h\sqrt{(zS)^2 + (\pi d_{\text{in}})^2}, \tag{5}$$

while the region of "smooth" flow has a contact area with the pipe surface

$$F_{\text{sm}} = F - F_{\text{vort}} = \pi d_{\text{in}}(zS) - 9h\sqrt{(zS)^2 + (\pi d_{\text{in}})^2}. \tag{6}$$

The expression for the density of the heat flux from the wall to the heat-transfer agent in a PTP takes the form

$$\dot{q}F = \dot{q}_{\text{I}} F_{\text{vort}} + \dot{q}_{\text{II}} F_{\text{sm}}, \tag{7}$$

where $\dot{q}_{\text{I}}$ and $\dot{q}_{\text{II}}$ are the densities of heat fluxes through the vortex and "smooth" zones, respectively.

We analyze the heat transfer in the vortex zone I by analogy with [8]. Here we assume that the velocity of the circulation flow near the wall has the order of the dynamic velocity $V^* = (\tau_w/\rho)^{0.5}$, while heat is transferred to the circulation flow, so that up to the time of separation of a vortex it is transferred to the external stream only by diffusion (a molecular mechanism) at the solid boundary. The distance over which heat diffuses in the circulatime time $\tau \approx (h/V^*)$ is detrmined by the expression

$$y \approx \sqrt{a\tau} \approx \sqrt{\frac{ah}{V^*}}. \tag{8}$$

Then the equation for the heat flux in the vortex zone I per unit area per unit time takes the form

$$\dot{q}_{\mathrm{I}} = \rho C_p (T_{\mathrm{w}} - T_s) \sqrt{\frac{aV^*}{h}} = \rho C_p \Delta T \sqrt{\frac{aV^*}{h}}. \tag{9}$$

We represent the heat flux in the region of "smooth" flow in the form [8]

$$\dot{q}_{\mathrm{II}} = \frac{\lambda}{y_h} (T_{\mathrm{w}} - T_s) = \frac{\lambda \Delta T}{y_h}. \tag{10}$$

Introducing the frictional stress $\tau_w = \mu(V^*/y_m)$ at the pipe wall, and taking $y_m \cong y_h$ [8], we obtain

$$\dot{q}_{\mathrm{II}} = \frac{\lambda \tau_{\mathrm{w}} \Delta T}{\mu V^*}. \tag{11}$$

The temperature T_s at the boundary between this region and the turbulent core of the stream is assumed to be the same as the analogous temperature for the vortex zone.

Using the functions (9) and (11), we find the density of the heat flux from the wall to the heat-transfer agent:

$$\begin{gathered} \dot{q} = \dot{q}_{\mathrm{I}} \frac{F_{\mathrm{vort}}}{F} + \dot{q}_{\mathrm{II}} \left(1 - \frac{F_{\mathrm{vort}}}{F}\right) = \\ = \rho C_p \Delta T \left\{ \sqrt{\frac{aV^*}{h}} \frac{9h}{\pi d_{\mathrm{in}}} \sqrt{1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2} + \frac{V^*}{\mathrm{Pr}} \left[1 - \frac{9h}{\pi d_{\mathrm{in}}} \sqrt{1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2}\right] \right\}. \end{gathered} \tag{12}$$

Thus, the total thermal resistance of the wall zone of a PTP, treated for the given model as a set of thermal resistances of the vortex region and the region of "smooth" flow connected in parallel, is

$$R_m = \frac{\Delta T}{\dot{q}} = \left\{ \rho C_p \left[\sqrt{\frac{aV^*}{h}} \frac{9h}{\pi d_{\mathrm{in}}} \sqrt{1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2} + \frac{V^*}{\mathrm{Pr}} \left(1 - \frac{9h}{\pi d_{\mathrm{in}}} \sqrt{1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2}\right) \right] \right\}^{-1}. \tag{13}$$

Substituting (13) into Eq. (3), we obtain the function determining the heat transfer in the flow of a one-phase heat-transfer agent in a PTP:

$$\begin{gathered} \mathrm{Nu}_{\mathrm{pr}} = \mathrm{Re\,Pr} \frac{\lambda_{\mathrm{pr}}}{8} \left\{ \mathrm{Pr}_t + \frac{(V^*)^2/V_f}{\left[\frac{9h}{\pi d_{\mathrm{in}}} \sqrt{1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2} \sqrt{\frac{aV^*}{h}} + \right.} \rightarrow \cdots \right. \\ \cdots \rightarrow \left. \frac{(V^*)^2/V_f}{\left. + \left(1 - \frac{9h}{\pi d_{\mathrm{in}}} \sqrt{1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2}\right) \frac{V^*}{\mathrm{Pr}} \right]} \right\}^{-1}. \end{gathered} \tag{14}$$

It is interesting to analyze this function in dimensionless form in comparison with the data for a smooth pipe. Since $V^* = (\lambda_{pr}/8)^{0.5} V_f$, and introducing the Reynolds number through the average flow velocity, $Re = (V_f d_{in})/\nu$, we finally obtain

$$\begin{gathered} \frac{\mathrm{Nu}_{\mathrm{pr}}}{\mathrm{Nu}_{\mathrm{sm}}} = \frac{\mathrm{Re\,Pr}\,\lambda_{\mathrm{pr}}}{8\mathrm{Nu}_{\mathrm{sm}}} \left\{ \mathrm{Pr}_t + \right. \\ + \frac{\mathrm{Re\,Pr}\,\lambda_{\mathrm{pr}}/8}{\frac{9}{\pi} \sqrt{\frac{h}{d_{\mathrm{in}}} \left[1 + \left(\frac{\pi d_{\mathrm{in}}}{zS}\right)^2\right]} \sqrt{\mathrm{RePr} \sqrt{\frac{\lambda_{\mathrm{pr}}}{8}}} + } \rightarrow \cdots \end{gathered} \tag{15}$$

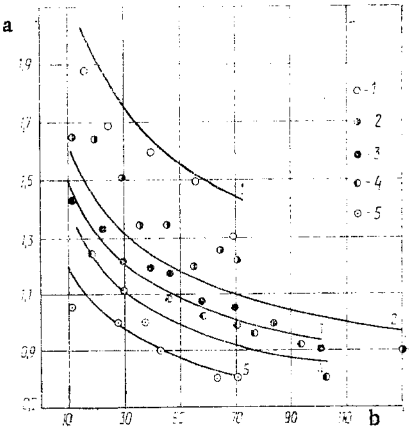

Fig. 2. Comparison of data on heat transfer during water flow in a PTP: 1) h = 0.69, S = 6.7, d_{in} = 17.0 mm; 2) 0.31, 6.5, 17.0; 3) 0.70, 14.0, 24.0; 4) 0.63, 20.0, 33.0; 5) 0.53, 20.0, 17.0; points) experimental data; solid lines) calculation from the function (15).

$$\cdots \rightarrow \frac{\mathrm{Re}\,\mathrm{Pr}\,\lambda_{pr}/8}{+\left[1-\frac{9h}{\pi d_{in}}\sqrt{1+\left(\frac{\pi d_{in}}{zS}\right)^2}\right]\mathrm{Re}\sqrt{\frac{\lambda_{pr}}{8}}}\Bigg\}^{-1}, \tag{15}$$

where

$$\mathrm{Nu}_{sm} = 0.023\ \mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.43};\quad \lambda_{pr} = \lambda_{sm}\left[1+13\left(\frac{hz}{t}\right)+94\left(\frac{hz}{t}\right)^2\right]$$

(see [9]). This function is valid for S > 9h.

According to [8], the turbulent Prandtl number Pr_t is a function of the Reynolds and Prandtl numbers. Since for water flow in a PTP one observes twisting of the stream, also influencing its core, the quantity Pr_t should also depend on the geometry of profiling of the pipe. Therefore,

$$\mathrm{Pr}_t = \mathrm{Pr}_t\,(\mathrm{Re},\ \mathrm{Pr},\ h,\ S,\ z,\ d_{in}). \tag{16}$$

The concrete form of this function was determined from a comparison of Eq. (15) with the authors' experimental data on heat transfer during water flow in a PTP,

$$\mathrm{Pr}_t = A\ \mathrm{Re}^m\mathrm{Pr}^n, \tag{17}$$

where A = 0.20; $m = 0.163 + 4.2(h/S^*)$; $S^* = \sqrt{(zS)^2 + (\pi d_{in})^2}$; n = 0.52. The expression obtained is valid to within 10% in the following ranges of the parameters:

$$h = (0{,}010 - 0.065)\,d_{in};\ S = (10 - 40)\,h;\ z = 3;$$
$$S^* = (56 - 107)_{mm};\ \mathrm{Re} = (10 - 120)\cdot 10^3.$$

A comparison of experimental data on a number of PTP with the results of calculations through the function (15) shows their satisfactory agreement (Fig. 2). We consider that the proposed semiempirical model of the process allows us to calculate heat transfer during water flow in profiled twisted pipes.

NOTATION

R, thermal resistance; λ_{pr}, effective coefficient of resistance; Pr_t, turbulent Prandtl number; V_s, stream velocity at the outer boundary of the region (zone) under consideration; V_f, average stream velocity; τ_w, wall shear stress; T_w, temperature of pipe wall; T_s, temperature of stream of heat-transfer agent at the outer boundary between the region and the turbulent core of the stream; $\dot{q}$, specific heat flux; a, coefficient of thermal diffusivity; τ, time; ρ, density of heat-transfer agent; C_p, specific heat of heat-transfer agent at constant pressure; λ, coefficient of thermal conductivity; y_h, thickness of the thermal boundary layer in the zone under consideration; y_m, thickness of the viscous sublayer; μ, coefficient of dynamic viscosity; t = zS, pitch of knurling (profiling) of the pipe along the filament; S, distance between adjacent projections (depressions); z, number of starts of profiling; h, height of a projection (depth of a depression); d_{in}, inside diameter of pipe along a smooth section; Nu, Re, Pr, St, Nusselt, Reynolds, Prandtl, and Stanton numbers, respectively. Indices: t, turbulent number (region of turbulent transfer); m, region of molecular transfer; w, wall; s, outer boundary of a region; f, average value; vort, vortex region; sm, region of "smooth" flow (smooth pipe); in, inside; pr, profiled pipe.

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ANALYSIS OF LONGITUDINAL VELOCITY FLUCTUATIONS ON A PLATE

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It is proposed to use the equation of the second moments with a mixing path length determined on the basis of experimental data for the longitudinal velocity fluctuations in a boundary layer.

In describing a number of hydrodynamics and heat-transfer problems, not only the averaged but also the fluctuation characteristics of the flow must be known. As an illustration, turbulent transfer processes in apparatus of chemical technology, high-temperature energetics, space and laser engineering can be cited. The heat flux in such apparatus depends not only on the turbulent transfer coefficients and the mean flow parameters, but also on the fluctuation structure of the flow, since it exerts substantial influence on the rate of physicochemical transformation and, consequently, on the heat and mass transfer.

Many paper [1-4], say, are devoted to the experimental investigation of fluctuating turbulent flow structure. Mainly problems of closing the turbulent transfer equations have been worked out theoretically [5-9]. The description and analysis of singularities in the velocity and temperature fluctuation distributions are limited.

An attempt is made in this paper to compute the longitudinal velocity fluctuation profile in the boundary layer on a plate around which a gradient-free gas flows. The problem of determining the average velocity has been studied sufficiently well for this case. The velocity profile is determined from the equations

$$\frac{\partial \rho \bar{u}}{\partial x} + \frac{\partial \rho \bar{v}}{\partial y} = 0; \tag{1}$$

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